Lecture 3: Dust dynamics and planetesimal formation

Having discussed the structure and evolution of the gaseous component of protoplanetary discs, we now turn our attention to the solid component of the disc. Solid bodies in discs can be split into three regimes, where different processes dominate their dynamics, and growth from sub-micron ISM grains to planetary sizes requires us to consider each of these size regimes. These regimes are as follows:

Dust
This catch-all term encompasses essentially all small solid particles, from sub-micron grains to objects with sizes of hundreds of metres[1]. The key physical distinction is that the dynamics of these particles is dominated by their interaction with the gas disc. For our purposes, the term "dust" will be used to describe any particles for which the effects of **aerodynamic drag** are important. (In practice, as we will see, this means sizes up to $\sim$ 1km.)

Planetesimals
These are bodies which are sufficiently large to interact with one another gravitationally – typically this means sizes $\gtrsim$10km (similar to small asteroids in the Solar System). Their dynamics is governed simply by gravity, and a population of planetesimals can be modelled via straightforward **N-body dynamics**. Consequently the processes by which planetesimals grow are relatively well understood, and this will be discussed in the next lecture.

Planetary cores / terrestrial planets
Once solid bodies grow to a mass $\sim$1$\mathrm{M}_{\oplus}$, they are massive enough to interact with the gas disc gravitationally. Torques between these bodies and the disc give rise to angular momentum exchange and **migration** through the disc, and sufficiently massive solid bodies ($\gtrsim$few $M_{\oplus}$) can also capture material from the disc and grow via **gas accretion**. These processes will be discussed in Lectures 5 & 4 respectively.

The first comprehensive model of planet formation was proposed by Safronov (1969), and although our understanding has evolved significantly in the intervening years, this picture still captures the essence of the process well. In this paradigm planet formation occurs in three stages, which roughly correspond to the three size regimes described above: i) collisional growth from dust to planetesimals; ii) formation of protoplanets from planetesimals; and iii) gas accretion or final assembly of terrestrial planets. Other mechanisms (such as gravitational instability) may well play a role, and the first stage – formation of planetesimals – remains particularly problematic, but this remains the most plausible model we have for the formation of most planetary systems. In this lecture we will look at the first stage, planetesimal formation, and consequently we must begin by considering the dynamics of dust particles.

1 Aerodynamic drag

Dust particles in protoplanetary discs are subject to gravity and centrifugal forces, and also feel aerodynamic drag from the disc gas. A spherical particle of radius s, moving at a velocity v relative to gas of density ρ_{g}, experiences an aerodynamic drag force which opposes its motion

$$F_{\mathrm{D}} = -\frac{1}{2} C_{\mathrm{D}} \pi s^2 \rho_{\mathrm{g}} v^2 \,. \tag{1}$$

This expression has three terms: the cross-sectional area of the grain, πs^2; the ram pressure exerted on the grain, $\rho_{\mathrm{g}} v^2$; and the drag coefficient C_{D}. In general the drag coefficient depends on the

[1]In the modern literature the terms "pebbles" or "boulders" are often used to describe particles of $\sim$cm-size or larger. However, these terms are not defined consistently, so for clarity we will largely avoid their use here.

velocity of the grain relative to the gas, and also on the size of the grain relative to the mean-free-path of gas molecules λ. Where the size of the particles is less than λ [formally where $s < (9/4)\lambda$], the effects of drag can be considered as the collective effects of collisions with individual molecules in the gas. This is known as Epstein drag, and in this regime the drag coefficient is given by

$$C_{\rm D} = \frac{8}{3}\frac{v_{\rm th}}{v}\,, \tag{2}$$

where $v_{\rm th} = (8/\pi)^{1/2} c_{\rm s}$ is the mean thermal velocity of the gas molecules. The drag force therefore scales linearly with the velocity v

$$F_{\rm D} = -\frac{4}{3}\pi s^2 v_{\rm th} \rho_{\rm g} v\,. \tag{3}$$

Larger particles instead interact with the gas as a fluid. This is known as the Stokes regime, and here the drag coefficient depends on the Reynolds number of the flow. In the Stokes regime $C_{\rm D}$ is usually approximated by a piecewise function of the Reynolds number, such as that given in Weidenschilling (1977), and the effects of drag forces are in general non-linear.

It is also useful to define the stopping timescale (or drag timescale), $t_{\rm s}$, which is the timescale on which frictional drag will cause an order-of-unity change in the momentum of the dust grain:

$$t_{\rm s} = \frac{mv}{|F_{\rm D}|}\,. \tag{4}$$

A dust particle has mass $m = (4/3)\pi\rho_{\rm d} s^3$ (where $\rho_{\rm d}$ is the material density of the dust), so in the Epstein regime the stopping time is therefore given by

$$t_{\rm s} = \frac{\rho_{\rm d}}{\rho_{\rm g}}\frac{s}{v_{\rm th}}\,. \tag{5}$$

We can re-write this in terms of the typical conditions in a protoplanetary disc at $\sim$AU radii

$$t_{\rm s} = 1\,{\rm s} \times \left(\frac{\rho_{\rm d}}{1\,{\rm g\,cm^{-3}}}\right)\left(\frac{\rho_{\rm g}}{10^{-9}\,{\rm g\,cm^{-3}}}\right)^{-1}\left(\frac{v_{\rm th}}{1\,{\rm km\,s^{-1}}}\right)^{-1}\left(\frac{s}{1\,\mu{\rm m}}\right) \tag{6}$$

Small dust grains have stopping times measured in fractions of a second, and are therefore extremely well coupled to the gas. However, metre-size particles have $t_{\rm s} \sim \Omega^{-1}$, and are only marginally coupled to the gas. Still larger bodies push into the Stokes regime, and the linear relationship between $t_{\rm s}$ and s breaks down. However, $t_{\rm s}$ is still an increasing function of size, and objects of km-size or larger have $t_{\rm s} \gg \Omega^{-1}$ and are essentially unaffected by gas drag.

2 Gas-dust dynamics

2.1 Settling

The simplest dynamical process we can consider is vertical settling of dust grains (sometimes referred to as sedimentation). For small grains the stopping timescale due to drag is negligibly short, so we can estimate the settling rate by equating the opposing forces of drag and gravity and computing the terminal velocity. As we saw in Lecture 2, for small vertical displacements ($z \ll R$) the vertical component of the gravitational force on a grain is

$$F_{\rm g} = -m\Omega_{\rm K}^2 z\,, \tag{7}$$

where $\Omega_{\rm K}$ is the Keplerian orbital frequency. We equate this with the drag force from Equation 3, and re-arrange to find the settling velocity (in the Epstein regime)

$$v_{\rm settle} = \frac{\Omega_{\rm K}^2}{v_{\rm th}}\frac{\rho_{\rm d}}{\rho_{\rm g}(z)} s z\,. \tag{8}$$

The settling timescale is therefore

$$t_{\rm settle} = \frac{z}{v_{\rm settle}} = \frac{v_{\rm th}}{\Omega_{\rm K}^2}\frac{\rho_{\rm g}(z)}{\rho_{\rm d}}\frac{1}{s}\,. \tag{9}$$

At $\sim$AU radii, $t_{\rm settle} \sim 10^5$yr for μm-size grains. However, the inverse scaling with s means that larger grains settle much more rapidly, and mm-size particles settle on timescales $\sim$ 100yr. Note also that the local gas density increases significantly towards the midplane [as $\rho_{\rm g} \propto \exp(-z^2/2H^2)$ in an isothermal disc]. Consequently, we expect dust to sediment rapidly out of the upper layers of the disc, and then settle more slowly as it approaches the midplane.

Observations of discs, however, tell us that settling is not this efficient. When we observe small grains we see clear evidence for settling and flattening of discs as they age (e.g., Furlan et al. 2006), but this is a weak effect and takes place over millions of years, not thousands. Larger mm-size grains do undergo strong settling (e.g., Villenave et al. 2020), as expected, but again this seems to occur on longer time-scales than predicted above. The reason that our simple analysis over-estimates the settling rate is that we have considered only a laminar disc, while real discs are turbulent. Settling of dust in a turbulent disc is a much more complex problem, as the turbulent motions can lift the grains to high z on relatively short timescales. In general the effect of turbulence is to drive diffusion of dust in the vertical direction, and this opposes the effect of vertical settling. Detailed models generally suggest that the vertical distribution of small grains in discs is sustained by a quasi-equilibrium between turbulent diffusion and sedimentation.

2.2 Radial drift

We now turn our attention to the effects of drag on the radial and azimuthal motions of dust grains. The crucial point to note is that solid bodies do not feel pressure forces, while the gas in the disc does. This simple fact gives rise to significant differential motion between the dust and gas, and makes gas drag the dominant factor in determining the motions of small particles[2].

The radial equation of motion (EoM) for gas in a thin disc is

$$\frac{v_{\phi,{\rm g}}^2}{R} = \frac{GM_*}{R^2} + \frac{1}{\rho_{\rm g}}\frac{dP}{dR}\,, \tag{10}$$

where P is pressure and $v_{\phi,{\rm g}}$ is the orbital speed of the gas. This equation is a balance of centrifugal acceleration, gravity and pressure, and if we neglect pressure we find that the gas orbits at the Keplerian speed $v_{\rm K} = \sqrt{GM_*/R}$. However, in general pressure is not negligible. The gas pressure in discs generally decreases with increasing radius (as both the density and temperature tend to be decreasing functions of radius). The pressure gradient therefore provides an additional outward force, and the gas orbits at slightly sub-Keplerian speeds. If we approximate the gas pressure at the midplane as a power-law $P \propto R^{-n}$, and adopt a locally isothermal equation of state $P = \rho_{\rm g} c_{\rm s}^2$, then the pressure gradient term in Equation 10 becomes

$$\frac{1}{\rho_{\rm g}}\frac{dP}{dR} = -n\frac{c_{\rm s}^2}{R}\,. \tag{11}$$

If we substitute this into Equation 10 and multiply by R, we find that

$$v_{\phi,{\rm g}}^2 = v_{\rm K}^2(1-\eta)\,, \tag{12}$$

[2]Note that the analysis presented here assumes dust particles to be "trace contaminants" which have no effect on one another or on the gas. Strictly this assumption holds only in the limit that the local dust-to-gas ratio $\rightarrow 0$. The dust "back-reaction" on the gas – which we have neglected here – is likely to be important in some circumstances, but for simplicity we do not consider this effect in detail.

where the term

$$\eta = n\frac{c_\mathrm{s}^2}{v_\mathrm{K}^2} \tag{13}$$

denotes how sub-Keplerian the gas is. The power-law index n depends on the disc's radial density and temperature profiles; typical values for viscous discs predict $n \simeq 2.75$–3. If $H/R = c_\mathrm{s}/v_\mathrm{K} = 0.05$ and $n = 11/4$ (consistent with a flaring disc with temperature profile $T \propto R^{-1/2}$), we see that the gas in a protoplanetary disc is typically sub-Keplerian by

$$(1-\eta)^{1/2} = 0.0034\,. \tag{14}$$

This is indeed a small number, and justifies our earlier assumption (in Lecture 2) that pressure is negligible in determining the radial structure of the gas disc. However, solid bodies are not subject to pressure forces, and therefore orbit at the Keplerian speed. Typical Keplerian orbital speeds at $\sim$AU radii are $\sim$10 km s^{-1}, so the gas is sub-Keplerian by $\sim$100 m s^{-1}. It is therefore obvious that solid bodies in such a disc experience a very strong headwind, and are subject to significant aerodynamic drag.

We now consider the motions of solid particles in the gas disc. Here the relevant timescale is the orbital period, so it is useful to define a dimensionless stopping time[3]

$$T_\mathrm{s} = t_\mathrm{s}\Omega_\mathrm{K} = t_\mathrm{s}\frac{v_\mathrm{K}}{R}\,. \tag{15}$$

The radial and azimuthal EoMs for a dust particle are

$$\frac{dv_{r,\mathrm{d}}}{dt} = \frac{v_{\phi,\mathrm{d}}^2}{r} - \Omega_\mathrm{K}^2 r - \frac{1}{t_\mathrm{s}}(v_{r,\mathrm{d}} - v_{r,\mathrm{g}}) \tag{16}$$

and

$$\frac{d}{dt}(rv_{\phi,\mathrm{d}}) = -\frac{r}{t_\mathrm{s}}(v_{\phi,\mathrm{d}} - v_{\phi,\mathrm{g}})\,. \tag{17}$$

The subscripts $_r$ and $_\phi$ denote the radial and azimuthal components of velocity respectively, with the additional subscripts $_\mathrm{g}$ and $_\mathrm{d}$ used to distinguish the gas and dust velocities. In the radial EoM, the first term is the centrifugal acceleration, the second term the acceleration due to gravity, and the third term the frictional drag force (which opposes the motion of the dust grain, and is zero if the dust and gas move at the same velocity). In the azimuthal direction the only acceleration is that due to the drag force; Equation 17 equates the rate of change of angular momentum with the drag torque. We make the simplifying assumption that the particles spiral inwards (or outwards) on nearly circular orbits, which is equivalent to assuming that the radial drift velocity is much smaller than the orbital velocity (i.e., $v_{r,\mathrm{d}} \ll v_{\phi,\mathrm{d}}$). To first order we can therefore write

$$v_{\phi,\mathrm{d}} \simeq v_{\phi,\mathrm{g}} \simeq v_\mathrm{K}\,. \tag{18}$$

This allows us to simplify the azimuthal EoM by noting that

$$\frac{d}{dt}(rv_{\phi,\mathrm{d}}) \simeq v_{\mathrm{r,d}}\frac{d}{dr}(rv_\mathrm{K}) = \frac{1}{2}v_{\mathrm{r,d}}v_\mathrm{K}\,. \tag{19}$$

We substitute this into Equation 17 and re-arrange to find

$$v_{\phi,\mathrm{d}} - v_{\phi,\mathrm{g}} \simeq -\frac{1}{2}\frac{t_\mathrm{s}v_\mathrm{K}}{r}v_{r,\mathrm{d}} = -\frac{1}{2}T_\mathrm{s}v_{r,\mathrm{d}}\,. \tag{20}$$

[3]The dimensionless stopping time T_s is also often known as the Stokes' number, denoted St. The two terms are generally used interchangeably, but be sure to check definitions when switching between different notation – extra factors of 2π are not uncommon.

The next assumption we make is that the net acceleration in the radial direction is negligible. This amounts to neglecting terms $O((H/R)^2)$, and allows us to set the LHS of Equation 16 to zero. If we re-write the Keplerian velocity in terms of the gas orbital velocity and η thus

$$\Omega_{\rm K}^2 r = \frac{v_{\phi,{\rm g}}^2}{r} + \frac{\eta v_{\rm K}^2}{r}\,, \qquad (21)$$

we can substitute for the appropriate term in the radial EoM to find

$$\frac{v_{\phi,{\rm d}}^2}{r} - \frac{v_{\phi,{\rm g}}^2}{r} - \eta\frac{v_{\rm K}^2}{r} - \frac{1}{t_{\rm s}}(v_{r,{\rm d}} - v_{r,{\rm g}}) = 0\,. \qquad (22)$$

We expand the first two terms as a difference of squares (noting that $v_{\phi,{\rm d}} + v_{\phi,{\rm g}} \simeq 2v_{\rm K}$), and substitute for $v_{\phi,{\rm d}} - v_{\phi,{\rm g}}$ from Equation 20. If we then divide by $v_{\rm K}/r$ and re-write in terms of $T_{\rm s}$, we find that

$$v_{r,{\rm d}}T_{\rm s} + v_{r,{\rm d}}T_{\rm s}^{-1} = v_{r,{\rm g}}T_{\rm s}^{-1} - \eta v_{\rm K}\,, \qquad (23)$$

and therefore the drift velocity of the dust is given by

$$v_{r,{\rm d}} = \frac{v_{r,{\rm g}}T_{\rm s}^{-1} - \eta v_{\rm K}}{T_{\rm s} + T_{\rm s}^{-1}}\,. \qquad (24)$$

This curve peaks at $T_{\rm s} = 1$. In the Epstein regime we have

$$T_{\rm s} = \frac{\rho_{\rm d}}{\rho_{\rm g}}\frac{s}{v_{\rm th}}\Omega_{\rm K}\,, \qquad (25)$$

so the drift velocity depends primarily on the particle size s. For typical parameters $T_{\rm s} = 1$ corresponds to $s \sim$ 10–100cm (and the Epstein regime remains valid up to $s \sim$ 1–10m). Smaller particles have short stopping times and remain well coupled to the flow, while large bodies are not strongly affected by gas drag. For particles near the peak of the curve (i.e., with $T_{\rm s} \sim 1$), however, gas drag produces a strong headwind torque which results in rapid inward radial drift. For a disc around $1{\rm M}_\odot$ star at 1AU, the drift velocity at the peak of the curve can be $\gtrsim 10{\rm m\,s^{-1}}$. At such velocities dust particles will spiral in to the central star in $\sim$ 100yr. If solid bodies are to grow to km-size or larger via collisions, they must be able to grow through this size range very rapidly; otherwise they will drift out of the disc within a few hundred orbits. This is commonly referred to as the "metre-size barrier", and how to overcome radial drift remains one of the major unsolved problems in planet formation.

A slight subtlety in this argument can be found in our assumption of an outward pressure gradient. This is true in the global sense, but need not necessarily be true locally. The equations of motion do not explicitly predict inward radial drift; rather, solid particles drift towards regions of high pressure[4]. If we are able to create local pressure maxima in the gas disc, either via small-scale structures or through turbulence, this may provide a means of overcoming the radial drift problem. Moreover, the accumulation of solid particles in local pressure maxima results in increased collision rates, and may lead to more rapid particle growth. This "trapping" of mm-sized particles in local pressure maxima has now been observed with ALMA (Dullemond et al. 2018), but it remains to be seen whether this process operates in the same way for larger particles.

[4]This is rather counter-intuitive, but can be easily understood. If we consider a local radial pressure maximum at radius R_0, the gas is sub-Keplerian for $R > R_0$ but super-Keplerian for $R < R_0$. Dust particles therefore feel a headwind at $R > R_0$, but receive a tailwind if $R < R_0$. The headwind torque removes angular momentum and causes inward drift, while the tailwind adds angular momentum and drives outward drift. The net effect is that dust migrates towards the pressure maximum from both directions.

3 Grain growth

For the smallest, sub-μm grains, inter-particle forces (usually van der Waals forces) generally result in efficient sticking when dust particles collide. If we begin with a population of ISM-like grains this can be well approximated by a simple "hit-and-stick" model of collisional growth. We have already seen that small grains are very well coupled to the gas by aerodynamic drag, so their velocity dispersion σ is approximately that of the Brownian motion of the particles

$$\sigma \simeq \sqrt{\frac{m_{\rm H}}{m}} c_{\rm s} \,. \tag{26}$$

If we assume that small particles stick with 100% efficiency when they collide, the mean growth rate of particle mass is simply given by

$$\frac{dm}{dt} = \pi s^2 \rho_{\rm g} Z \sigma \,. \tag{27}$$

Here Z is the dust-to-gas ratio in the disc, and as before we assume that the grains are spherical. The mass of a grain $m = (4/3)\pi s^3 \rho_{\rm d}$, so the rate of growth is

$$\frac{ds}{dt} = \frac{\rho_{\rm g}}{4\rho_{\rm d}} Z \sigma \,. \tag{28}$$

If we assume a canonical dust-to-gas ratio of $Z = 0.01$ and again assume typical parameters for $\sim$AU radii ($\rho_{\rm g} \sim 10^{-9}$g cm^{-3}, $c_{\rm s} \simeq 1$km s^{-1}), we find that the growth rate at the midplane is

$$\frac{ds}{dt} \simeq 10^{-4} \text{cm yr}^{-1} \,. \tag{29}$$

In practice Brownian motion sets a lower limit to the velocity dispersion of the grains, so both σ and ds/dt are likely to be significantly larger than this crude estimate suggests. (The assumption of Brownian motion is accurate only for grains with $s \lesssim 0.1\mu$m.) Collisional growth is therefore extremely efficient, and even if grains do not stick with 100% efficiency we still expect growth to mm- or cm-size to occur on a timescale $\lesssim 10^4$yr.

Unfortunately at this point several of our assumptions break down. Gas drag starts to become important, and the rate of radial drift increases substantially (as does the velocity dispersion). The particles also do not remain spherical as they grow; small monomers instead agglomerate in a random fashion, and the resulting aggregates are fractal in nature and very porous. Moreover, cm-size particles simply do not stick together when they collide[5]. Collisional growth of particles has been studied extensively via both numerical simulations and laboratory experiments, and remains an active area of research. (See the reviews by Blum & Würm 2008, Johansen et al. 2014, and Drążkowska et al. 2023, for a detailed discussion.) Several uncertainties remain, but the broad consensus is that collisional growth is efficient up to sizes in the mm to cm range. However, collisions of cm-size are more likely to result in some combination of bouncing, compaction, and shattering (depending on the collision velocities and the porosity of the particles), and collisional growth essentially stalls at this point.

4 Planetesimal formation

By considering the effects of aerodynamic drag and particle collisions we have identified two major hurdles to forming planetesimals: radial drift, and the inefficiency of collisional growth. Both of these become important when particles reach roughly cm-size, and it is extremely difficult to

[5]To paraphrase Doug Lin, if I throw rocks at other rocks, the one thing I won't ever get is bigger rocks!

grow particles to larger sizes gradually. Forming km-size planetesimals must presumably proceed via some other mechanism, but despite a plethora of ideas this remains an unsolved problem. Significant progress has been made in recent years, however, and here we discuss some of the most promising lines of investigation.

4.1 The Goldreich-Ward mechanism

The so-called Goldreich-Ward instability (Goldreich & Ward 1973) is the basic idea behind several modern theories of planetesimal formation. In this scenario vertical settling of small grains steadily increases the dust-to-gas ratio at the disc midplane, until eventually the dust layer becomes gravitationally unstable and fragments. We can estimate the conditions at which this occurs by a simple application of the Toomre (1964) criterion, setting

$$Q = \frac{\sigma\Omega}{\pi G \Sigma_{\rm dust}} = 1\,, \tag{30}$$

where $\Sigma_{\rm dust}$ is the surface density of the dust layer. For a canonical dust-to-gas ratio of 1:100 and a gas surface density of 100–1000 $\mathrm{g\,cm^{-2}}$, a velocity dispersion of $\sigma \lesssim 10\,\mathrm{cm\,s^{-1}}$ is required in order for gravitational instability to fragment the dust layer at $\sim$AU radii. This is much, much less than the typical sound speed in the gas ($\sim 1\,\mathrm{km\,s^{-1}}$), so the dust layer must become extremely thin in order for the dust layer to become unstable. The typical fragment mass is

$$M_{\rm frag} \sim \Sigma_{\rm dust} H_{\rm dust}^2 = \Sigma_{\rm dust}\left(\frac{\sigma}{\Omega}\right)^2 . \tag{31}$$

For the parameters above gives $M_{\rm frag} \sim 10^{16}$g, which corresponds to planetesimals a few km in radius. Gravitational fragmentation proceeds on the dynamical time-scale, so the Goldreich-Ward mechanism in principle allows km-size planetesimals to form very rapidly from small, mm- to cm-size bodies. This potentially provides a means of overcoming the problems of both radial drift and sticking, and is consequently a very attractive model for planetesimal formation.

Unfortunately we can easily show that, in its simplest form, the Goldreich-Ward mechanism does not result in planetesimal formation. The instability requires that the dust layer be approximately 10^4 times thinner than the gas disc, and consequently at the disc midplane the local dust density dramatically exceeds the local gas density (by a factor ~ 100). The dust therefore dominates the local dynamics, and the dust and gas both orbit at the Keplerian speed. Above this layer, however, the gas is significantly sub-Keplerian due to gas pressure, and there is a large velocity shear in the vertical direction. This shear is Kelvin-Helmholz unstable, and the resulting turbulence prevents σ from becoming small enough for instability to set in (e.g., Cuzzi et al. 1993). More generally, any turbulence in the gas limits the efficiency of vertical settling, and in real discs it seems unlikely that the dust layer will ever become thin enough for the Goldreich-Ward instability to operate.

4.2 Planetesimal formation in turbulent flows

Realistic models of planetesimal formation require us to consider the various processes of grain growth against the background of a turbulent protoplanetary disc. The evolution is in general non-linear, and modelling it requires large-scale numerical simulations. However, we can gain some qualitative behaviour understanding by considering the results we have discussed above. Dust particles tend to migrate towards local pressure maxima, and "trapping" of dust particles in the turbulence can significantly enhance the rate of collisional growth. However, the velocity dispersion in the turbulent gas is a significant fraction of the sound speed (as discussed in Lecture 2), and this in turn excites relatively high collision velocities between solid bodies. For typical disc parameters we find that "pebbles" and "boulders" (i.e., cm- to m-size bodies) have typical collision speeds of tens of $\mathrm{m\,s^{-1}}$, which are much too large for sticking (and usually result in shattering/fragmentation).

Turbulence alone is therefore not enough to drive planetesimal formation, as the benefits of increased particle concentration are offset by the much higher collision speeds.

A more promising line of investigation is the discovery of two-fluid instabilities, such as the so-called streaming instability (Youdin & Goodman 2005). Such instabilities are driven by the velocity difference between dust and gas, and the basic premise of the streaming instability is simple. Solid bodies move with respect to the gas, and consequently feel a headwind drag force. However, if solids clump together in sufficiently large concentrations the drag force is reduced (as the solids are shielded from the headwind). This in turn leads to further concentration of particles (as large clumps decouple from the gas and "trap" more solid bodies) and further reduction in the headwind drag, and the process rapidly runs away. Eventually the particle clumps become self-gravitating, and can collapse to form planetesimals.

Numerical simulations have shown that this process can lead to the formation of large, ~ 100 km-size planetesimals on dynamical timescales (e.g., Johansen et al. 2007), but large uncertainties still remain in these calculations. The development of the streaming instability is very sensitive to the local dust-to-gas ratio, and it may not operate efficiently in some (most?) protoplanetary discs. Moreover, the instability is most efficient for bodies with $T_{\rm s} \sim 1$ (i.e., $s \sim$ 10–100cm), and it is not clear that collisional growth is sufficient to produce large numbers of bodies in this size regime. It seems likely both streaming instabilities and turbulence play an important role in the growth of solid bodies, but a full understanding of planetesimal formation still eludes us. (For detailed reviews of planetesimal formation in turbulent discs, see Chiang & Youdin 2010, Johansen et al. 2014, and Drążkowska et al. 2023).

More recently, a similar process that has gained much attention is so-called "pebble accretion". First proposed by Lambrechts & Johnsen (2012), in this picture "pebbles" (particles with $T_{\rm s} \sim 1$) preferentially accrete on to any bodies massive enough to interact with the gas gravitationally. Usually this means planetary cores of ∼Earth mass or larger, but this process can be efficient even for large planetesimals. Gas accretes on to such bodies via tidal streams, but material passing through the planet's Hill sphere must lose energy and/or angular momentum in order to remain bound. Gas accretion in this regime is often inefficient (see Lecture 4), but particles which are weakly coupled to the gas (i.e., particles with $T_{\rm s} \sim 1$) quickly spiral inwards due to gas drag. Pebbles can be accreted very efficiently in this manner, so this mechanism rapidly enriches the forming planet in solids. Some of the same criticisms mentioned above apply here also (in particular, planetesimals or planetary cores must already have formed in order to "trigger" this process), but pebble accretion offers very promising means of growing massive solid bodies quickly, potentially solving the core formation time-scale problem that has long been a major stumbling block in our picture of planetary growth.

Further Reading

In addition to the main list of references given on the course home-page, the following papers are particularly relevant to this lecture:

Blum & Wurm, *The Growth Mechanisms of Macroscopic Bodies in Protoplanetary Disks*, 2008, ARA&A, 46, 21.
Chiang & Youdin, *Forming Planetesimals in Solar and Extrasolar Nebulae*, 2010, AREPS, 38, 493.
Youdin, *From Grains to Planetesimals: Les Houches Lecture*, `arXiv:0807.1114`.
Goldreich & Ward, *The Formation of Planetesimals*, 1973, ApJ, 183, 1051.
Johansen et al., *Rapid planetesimal formation in turbulent circumstellar disks*, 2007, Nature, 448, 1022.
Cuzzi et al., *Particle-gas dynamics in the midplane of a protoplanetary nebula*, 1993, Icarus, 106, 102.
Youdin & Goodman, *Streaming Instabilities in Protoplanetary Disks*, 2005, ApJ, 620, 459.
Lambrechts & Johansen, *Rapid growth of gas-giant cores by pebble accretion*, 2012, A&A, 544, 32.
Dullemond et al., *The Disk Substructures at High Angular Resolution Project (DSHARP). VI. Dust Trapping in Thin-ringed Protoplanetary Disks*, 2018, ApJ, 869, L46.
Furlan et al., *A Survey and Analysis of Spitzer Infrared Spectrograph Spectra of T Tauri Stars in Taurus*, 2006, ApJS, 165, 568.
Villenave et al., *Observations of edge-on protoplanetary disks with ALMA. I. Results from continuum data*, 2020, A&A, 642, A164.
Johansen et al., *The multi-faceted planetesimal formation process*, 2014, Protostars & Planets VI, p547 (`arXiv:1402.1344`).
Bae et al., *Structured Distributions of Gas and Solids in Protoplanetary Disks*, 2023, Protostars & Planets VII, `arXiv:2210.13314`
Drążkowska et al., *Planet Formation Theory in the Era of ALMA and Kepler: from Pebbles to Exoplanets*, 2023, Protostars & Planets VII, `arXiv:2203.09759`